Fuzzy Control Model Optimization for Behavior-Consistent Traffic Routing Under Information Provision

Alexander Paz and Srinivas Peeta

Abstract—**This paper presents an H-infinity filtering approach to optimize a fuzzy control model used to determine behavior-consistent (BC) information-based control strategies to improve the performance of congested dynamic traffic networks. By adjusting the associated membership function parameters to better respond to nonlinearities and modeling errors, the approach is able to enhance the computational performance of the fuzzy control model. Computational efficiency is an important aspect in this problem context, because the information strategies are required in subreal time to be real-time deployable. Experiments are performed to evaluate the effectiveness of the approach. The results indicate that the optimized fuzzy control model contributes in determining the BC information-based control strategies in significantly less computational time than when the default controller is used. Hence, the proposed H-infinity approach contributes to the development of an efficient and robust information-based control approach.**

Index Terms—**Fuzzy control, H-infinity filter, information-based control.**

I. INTRODUCTION

FUZZY control through a rule-based mechanism can be used to determine behavior-consistent (BC) information-based control strategies for route guidance to robustly respond to the performance enhancement objectives of a system controller in a dynamic vehicular traffic system. BC strategies explicitly factor the likely driver response behavior to information provision in determining the controller-proposed route guidance strategies [1]. In this context, fuzzy control provides a convenient mathematical handle to treat the uncertainty and vagaries associated with human decision making in general and, additionally, the subjective interpretation of the linguistic attributes of the information provided. In previous work [1], [2], the authors showed the effectiveness of their fuzzy control model to determine BC information-based control strategies. The fuzzy control model defines a fuzzy system that continuously seeks real-time information-based control strategies to improve the overall vehicular traffic system performance.

The performance of a fuzzy system depends on its rule base and membership functions. The rule base is a collection of relations in the form of *if–then* rules that are used to determine the control actions based on the current and desired system states. Mathematically, the membership functions are used to represent and handle the vagueness of inputs and their associated consequences on the outputs; and the operations of fuzzy sets facilitate the building of the logical framework for reasoning with variables that are vague in nature, such as language-based descriptors (e.g., congestion ahead, the error is negative large).

Given a rule base, the membership functions can be trained (optimized) to enhance the performance of the fuzzy system. The initial membership functions can be constructed based on experience, and they can later be trained to capture nonlinearities and modeling errors. Hence, trained membership functions result in better computational performance, because the system can better respond to the nonlinearities and modeling errors. In the context of information-based real-time control of vehicular traffic systems, both nonlinearities and modeling errors are significant elements. Nonlinearities are a consequence of the intricate network-level interactions and dynamics as well as the human behavior component. Modeling errors are mainly a reflection of the imperfections and/or limitations underlying the driver behavior models, traffic flow models and the relationships between the control inputs and outputs. Nonlinearities and modeling errors are always present, given the complexity of human behavior and the associated consequences on network interactions and dynamics.

The methods for training fuzzy membership functions can be broadly divided into derivative-based or derivative-free methods [3]. Derivative-based methods include, among others, the gradient descent, Kalman filtering, simplex method, least squares, and backpropagation. Derivative-free methods range from genetic algorithms to heuristic methods.

H-infinity and Kalman filters are derivative-based methods that can be used to estimate and optimize system states that cannot be directly observed. The Kalman filter works well only under certain characteristics for the noise process. The process and measurement noises in the system need to have zero means. This zero-mean property must hold at each time instant and across the entire time history of the process. That is, the expected value of noise at each time instant must be equal to zero. From a data-need standpoint, the Kalman filter requires knowledge of the distribution of the noise processes and uses the associated covariance matrices as design parameters. The attractiveness of the Kalman filter is that it is the minimum variance estimator; it results in the smallest possible standard deviation of the estimation error.

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The H-infinity (minimax) filter assumes that nature is throwing the worst possible noise at the estimator, which can be limiting if untrue. Thereby, it minimizes the worst case estimation error. The optimization of fuzzy membership function parameters involves high levels of nonlinearities; the H-infinity filter has been shown to provide better results than the Kalman filter in this context [4], [5]. This is because the H-infinity filter is often more robust to system noise, modeling errors, and nonlinearities than the Kalman filter [4], [5]. In our problem context, these characteristics make the H-infinity filtering approach suitable for training the fuzzy membership functions that are used by an information-based fuzzy control model to improve the performance of a vehicular traffic system.

Given the difficulties in obtaining data on the covariance matrices for the Kalman filter in our context and the more restrictive nature of its assumptions for the noise processes, this paper proposes an H-infinity filtering approach to optimize the proposed fuzzy control model. By adjusting the membership function parameters to minimize the worst case estimation error to better respond to nonlinearities and modeling errors, the approach is able to enhance the computational performance of the fuzzy control model. Results from experiments suggest that the optimized fuzzy control model determines the BC information-based control strategies using significantly less computational time than the default fuzzy control model. This is synergistic with the real-time route guidance problem, as information strategies are required in subreal time to be deployable.

The remainder of this paper is organized as follows. Section II describes the problem. Section III expands the initial formulation of the fuzzy control model [1] to explicitly identify the parameters to optimize. Section IV discusses the H-infinity methodology proposed to address the problem. Section V describes the study experiments and analyzes the results. Section VI presents some concluding comments.

II. Problem Description

The problem addressed here focuses on optimizing the computational efficiency of a fuzzy control model in which a controller seeks to improve the performance of a vehicular traffic system via real-time BC information provision to drivers. It involves the optimization of the membership function parameters used by the controller to determine the control strategies. This leads to a faster convergence to the control strategies to be provided to drivers.

Fig. 1 illustrates the overall logic of the broader problem, i.e., the BC real-time route guidance under information provision, of which the problem addressed in this paper is a subproblem. Addressed in a real-world deployment context, it uses a rolling horizon stage-based approach to control the system in real time for a predetermined planning horizon. At each stage σ, the current traffic network conditions and the projected origin–destination (O-D) demand for the next stage are used to generate the system optimal (SO) dynamic traffic assignment solution for the next stage. The controller uses these SO route assignment proportions and an iterative procedure involving a controller-estimated model of driver behavior and the fuzzy control model, to generate BC routing strategies to provide route guidance to drivers [1], [2], so that the actual driver decisions in the next stage result in close to the SO route proportions. The offline parametric optimization of the fuzzy control model to enhance online computational efficiency is the focus of this paper. The fuzzy control model is highlighted by the nonshaded box in the figure. The rest of the Fig. 1 is given as follows: The controller provides the driver routes in the next stage. The drivers use this information and their innate behavioral tendencies to make pretrip and/or en-route routing decisions. The loop is completed when the driver decisions and the associated traffic flow interactions lead to the unfolding field traffic conditions measured through sensors.

The fuzzy control model aims to determine the proportions of drivers that should be recommended specific routes and/or the set of linguistic messages describing route conditions, so that, when drivers make their decisions according to the controller-estimated driver behavior model, the actual route proportions are close to the SO solution route proportions. It does so by minimizing the error between the SO proportions and the route proportions obtained from the estimated driver behavior model. Thereby, the information strategies provided as input to the estimated driver behavior model are the outcome of the minimization of this error. Fig. 2 illustrates the details of the fuzzy control model, which are discussed in Section III.

While not a focus of this paper, it should be mentioned that the SO solution is used here as a “desired goal” by the controller, who seeks to steer the traffic network as close as possible to the time-dependent SO solution by providing BC information. For individual drivers, this implies recommending routes that they would normally consider (even in the absence of information) that also coincide with the SO routes, to the extent possible.

The broader problem illustrated in Fig. 1 is described in detail in [1], whereas the fuzzy control model shown in Fig. 2 is addressed in [2]. This paper addresses the fuzzy control model optimization to enhance computation efficiency, which is represented by the nonshaded boxes of the flowchart in Fig. 4.

III. Fuzzy Control Model

Fig. 2 shows the iterative search procedure involving the controller-estimated driver behavior model and the fuzzy control model to determine the BC information strategies to be provided to drivers for an O-D pair. While this search occurs within a rolling horizon stage σ, as shown in Fig. 1, the time dimension associated with the discrete time intervals within this stage is ignored hereafter without loss of generality to simplify the notation. The search procedure seeks to minimize error e between the SO proportions and the proportions predicted by the controller-estimated driver behavior model. The fuzzy control model, as illustrated by the nonshaded boxes in the figure, represents the search mechanism. Here, n represents the iteration counter for the search, which terminates if the errors for all controllable routes k for that O-D pair are within a threshold value. Controllable routes are defined as routes that are both SO routes (also labeled controller-desired routes) and driver preferred (routes that a driver would consider even in the absence of information). It is possible that some O-D pairs may

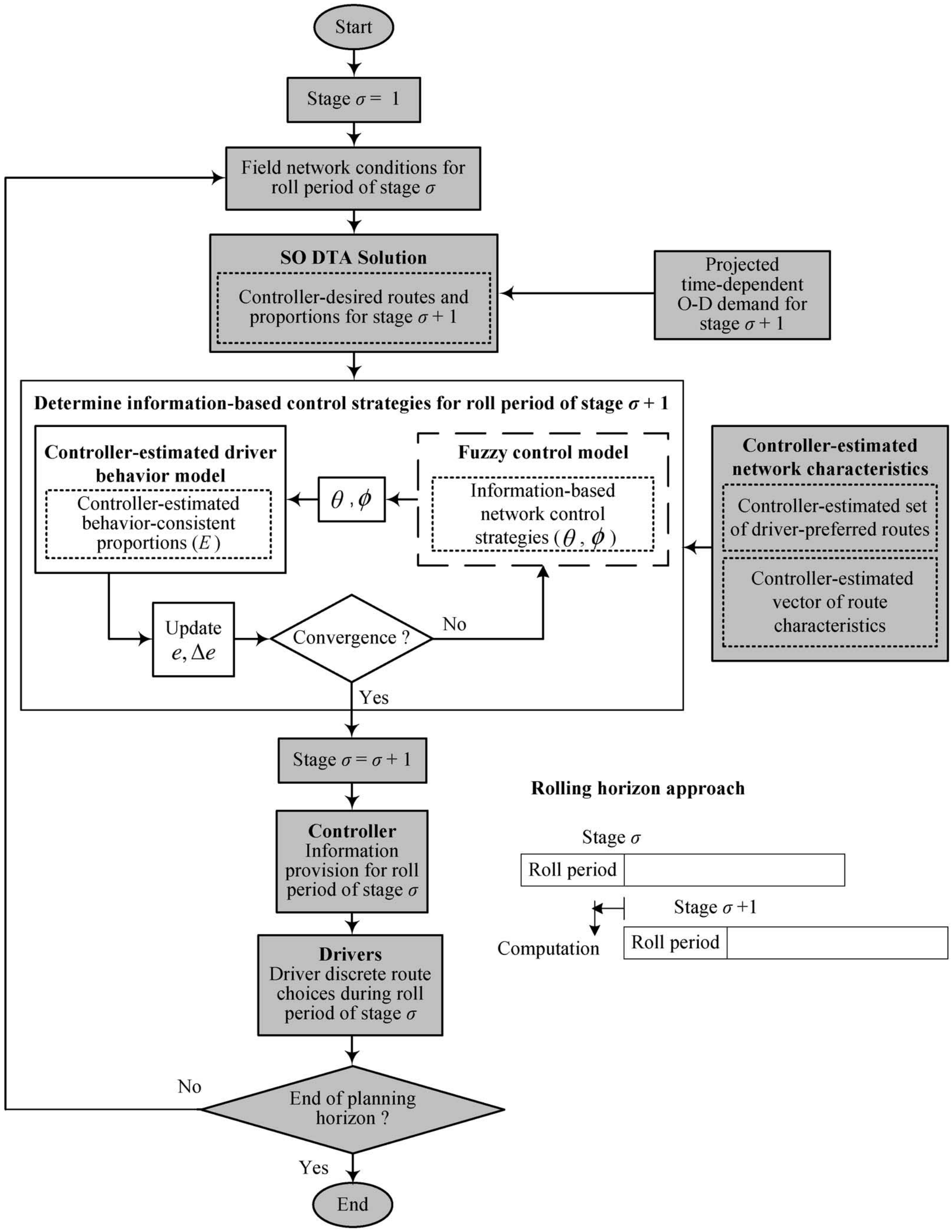

Fig. 1. Conceptual framework for the BC real-time traffic routing problem under information provision.

have zero controllable routes, in which case no search is done for them.

As shown in Fig. 2, the fuzzy control model consists of an input stage (dotted box), a decision processing stage (solid boxes), and an output stage (dashed box). The inputs are the vectors of errors and error changes Δe. The processing stage uses a set of *if–then* rules and standard fuzzy logic procedures [6] to determine the degrees of change or adjustment to the information strategies (θ, ϕ) from the previous iteration. For example, the first *if–then* rule used by the fuzzy control model (in Fig. 2) states the following: *if* error e is negative small and error change Δe is positive large, *then* change information strategy θ by a positive small (PS) quantity $(\Delta\theta)$. Table I shows the set of *if–then* rules used. A detailed description of these rules is provided in [1]. The output stage uses the degrees of adjustment to determine the updated information strategies to be provided to the controller-estimated driver behavior model in the next iteration. The fuzzy control model is described in detail hereafter to illustrate its relationship to the H-infinity optimization approach proposed in this paper.

For a given O-D pair and a stage σ, the objective of the fuzzy control model is to minimize, over the set of controllable routes for that O-D pair, the difference between the SO route proportions (which are constant for the roll period of a stage) and the controller-estimated proportions of drivers taking routes.

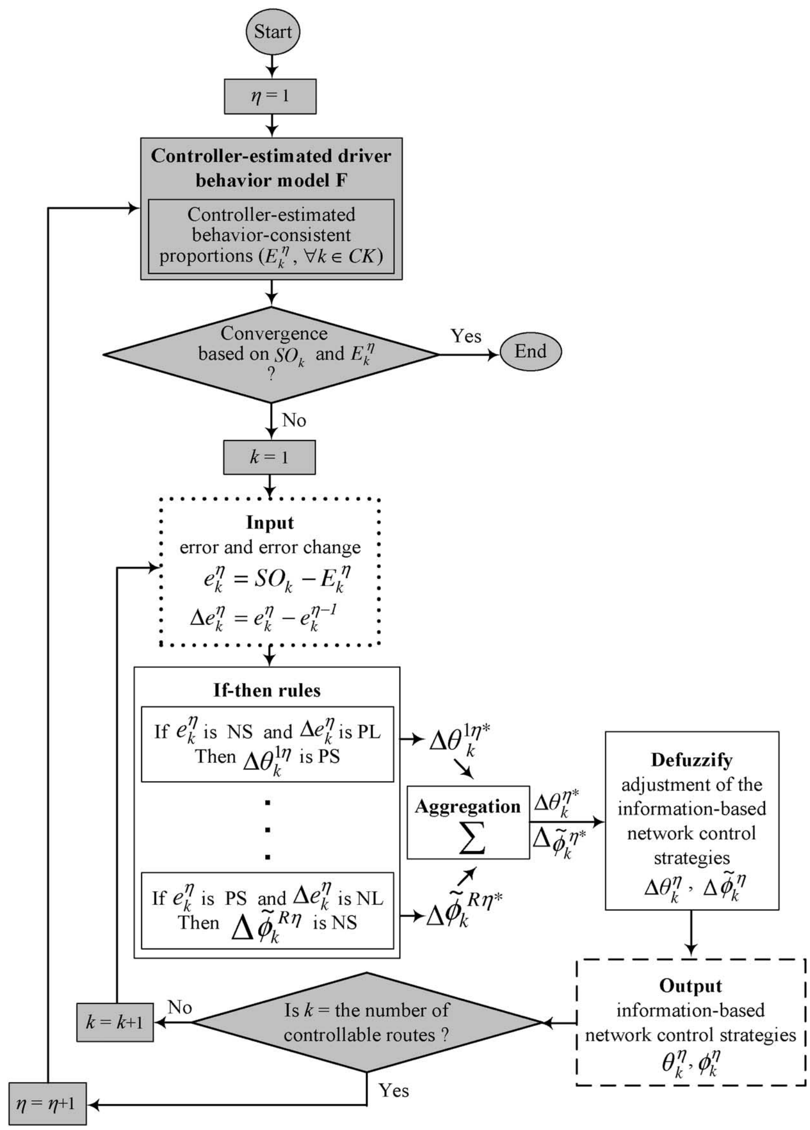

Fig. 2. Iterative search procedure to determine the BC information-based control strategies.

TABLE I
IF–THEN RULES FOR PRESCRIPTIVE AND DESCRIPTIVE INFORMATION

		Error (e) NL	NS	ZR	PS	PL
Change in Error (Δe)	NL	NL	NL	NS	NS	NS
	NS	NL	NS	ZR	ZR	ZR
	ZR	NL	NS	ZR	PS	PL
	PS	ZR	ZR	ZR	PS	PL
	PL	PS	PS	PS	PL	PL

where: NL = Negative large
NS = Negative small
ZR = Zero
PS = Positive small
PL = Positive large

A. Fuzzy Control Inputs

The inputs to the fuzzy control model are the vectors of error e_k^n and change in error Δe_k^n defined by

$$e_k^n = \mathrm{SO}_k^{\rho(\sigma)} - E_k^{\rho(\sigma)n} \quad \text{and} \quad \Delta e_k^n = e_k^n - e_k^{n-1} \tag{1}$$

where e_k^n is the difference between the SO proportions $\mathrm{SO}_k^{\rho(\sigma)}$ for route k and the controller-estimated proportions of the drivers taking route k for roll period ρ of stage σ, $E_k^{\rho(\sigma)n}$ (which are obtained using the information-based control strategies computed for iteration n). The change in error Δe_k^n is the difference between the error in current iteration n, e_k^n, and the error in the previous iteration $n-1$, e_k^{n-1}.

B. Membership Functions

Fig. 3 shows an example of the sets of membership functions associated with the *if–then* rules. The top part of the figure depicts the default membership functions, which are identical for all O-D pairs. They are designed to evenly cover the domain of the input and output variables. This is a common and convenient approach used in fuzzy control, as the default parameters are sufficient to determine the control strategies. That is, the optimization is required not to achieve accuracy in fuzzy control but to influence the computational performance. The bottom part depicts an example of an optimized set of

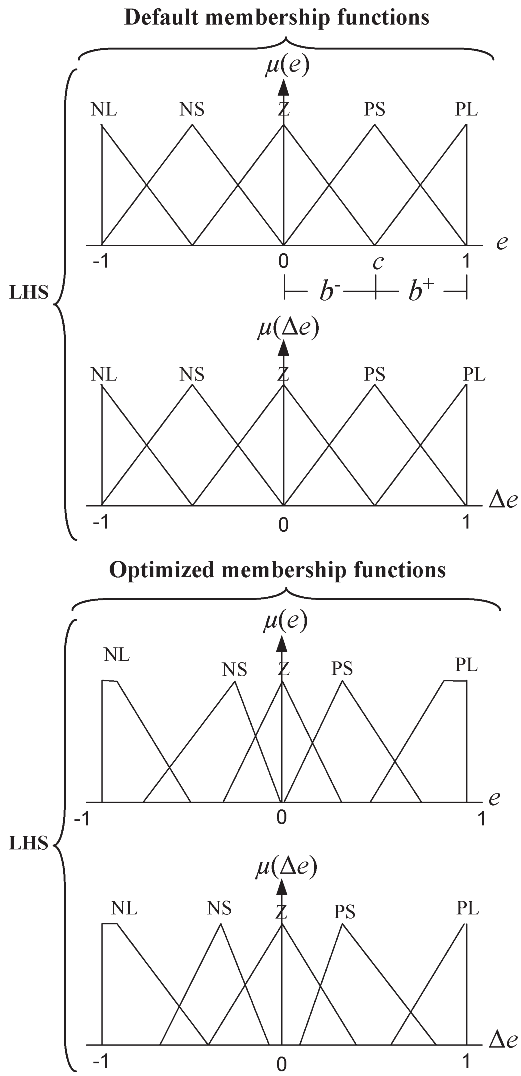

Fig. 3. Example of default and optimized membership functions.

membership functions for a specific controllable route for an O-D pair.

Consider the gth fuzzy membership function of the fth input z_{kf} for route k (here, $z_{k1} = e_k^n$, and $z_{k2} = \Delta e_k^n$). Its modal point, lower half-width, and upper half-width are denoted as c_{kgf}, b_{kgf}^-, and b_{kgf}^+, respectively. The membership function is equal to 1 when the input is c_{kgf}. As the input increases or decreases from c_{kgf}, the membership function value linearly decreases to 0 at $c_{kgf} + b_{kgf}^+$ and $c_{kgf} - b_{kgf}^-$, respectively. For input values less than $c_{kgf} - b_{kgf}^+$ or higher than $c_{kgf} + b_{kgf}^+$, the membership function is equal to 0. Hence, the degree of membership of the fth crisp input for route k, z_{kf}, in its gth fuzzy set is given by

$$\mu_{kgf}(z_{kf}) = \begin{cases} 1 + (z_{kf} - c_{kgf})/b_{kgf}^-, & \text{if } -b_{kgf}^- \leq (z_{kf} - c_{kgf}) \leq 0 \\ 1 - (z_{kf} - c_{kgf})/b_{kgf}^+, & \text{if } 0 \leq (z_{kf} - c_{kgf}) \leq b_{kgf}^+ \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

The fuzzy control model has two outputs for each route k: the prescriptive (θ_k) and descriptive (ϕ_k) information strategies. Prescriptive information corresponds to route recommendations for individual drivers, whereas descriptive information corresponds to qualitative descriptions of routes (linguistic variables) provided through mass dissemination devices. Although the descriptive information variable is defined by discrete fuzzy sets in reality (for example, "congestion ahead" or "heavy traffic"), it is viewed as the outcome of continuous crisp values in our approach. Its representation through intermediate continuous variables, $\widetilde{\phi}_k^n$, is used to achieve smooth convergence and reduce jumps in the objective function resulting from switching between discrete linguistic messages.

The numbers of rules corresponding to the prescriptive and descriptive information are RP and RG, respectively. Hence, there are a total of $R = RP + RG$ rules in our fuzzy system. The consequent of the hth rule for output w for route k is a triangular fuzzy set with modal point d_{khw}, lower half-width β_{khw}^-, and upper half-width β_{khw}^+. Hence, the fuzzy set of the consequent of the hth rule for k for output w is given by (3), where w takes a value of 1 for descriptive information and a value of 2 for descriptive information

$$\mu_{khw}(y) = \begin{cases} 1+(y-d_{khw})/\beta_{khw}^-, & \text{if } -\beta_{khw}^- \leq (y-d_{khw}) \leq 0 \\ 1-(y-d_{khw})/\beta_{khw}^+, & \text{if } 0 \leq (y-d_{k,hw}) \leq \beta_{khw}^+ \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

C. Decision Process

The fuzzy control model uses the Larsen product implication operator for fuzzy inference and approximate reasoning, the max–min composition operator for aggregation, and the center of gravity method for defuzzification (see [6] for details).

The current inputs e_k^n and Δe_k^n are matched against all the R defined set of rules, resulting in RP fuzzy sets for adjustment $\Delta\theta_{kh}^{n^*}, h = 1, \ldots, RP$, of θ_k and RG fuzzy sets for adjustment $\Delta\widetilde{\phi}_{kh}^{n^*}, h = RP+1, \ldots, R$, of ϕ_k. The degree at which rule h is activated, which is denoted by γ_{kh}^n, depends on the relevant components of e_k^n and Δe_k^n, as given by

$$\gamma_{kh}^n = \max_{z \in Z} \min\left(\mu_{e_k^n}(z), \mu_{\Delta e_k^n}(z)\right) \quad (4)$$

where Z represents the universe of discourse for fuzzy sets e_k^n and Δe_k^n. The Larsen implication operator is used to apply approximated reasoning and obtain the membership functions of the outcomes $\Delta\theta_{kh}^{n^*}$ and $\Delta\widetilde{\phi}_{kh}^{n^*}$ defined as

$$\mu_{\Delta\theta_{kh}^{n^*}} = \gamma_{kh}^n \cdot \mu_{\Delta\theta_{kh}^n} \text{ and } \mu_{\Delta\widetilde{\phi}_{kh}^{n^*}} = \gamma_{kh}^n \cdot \mu_{\Delta\widetilde{\phi}_{kh}^n}. \quad (5)$$

For each route k, the following scheme is used to aggregate the outcomes from all rules:

$$\mu_{\Delta\theta_k^{n^*}} = \sum_{h=1}^{RP} \mu_{\Delta\theta_{kh}^{n^*}} \text{ and } \mu_{\Delta\widetilde{\phi}_k^{n^*}} = \sum_{h=RP+1}^{R} \mu_{\Delta\widetilde{\phi}_{kh}^{n^*}}. \quad (6)$$

Once the fuzzy aggregate outcomes $\Delta\theta_h^{n^*}$ and $\Delta\widetilde{\phi}_h^{n^*}$ are defined, they are defuzzified using the center of gravity method given by

$$\Delta\theta_k^n = \frac{\sum_{h=1}^{RP} \overline{\theta}_{kh} \cdot S(\mu_{\Delta\theta_{kh}^{n*}})}{\sum_{h=1}^{RP} S(\mu_{\Delta\theta_{kh}^{n*}})} \tag{7}$$

$$\Delta\widetilde{\phi}_k^n = \frac{\sum_{h=RP+1}^{R} \overline{\phi}_{kh} \cdot S(\mu_{\Delta\widetilde{\phi}_{kh}^{n*}})}{\sum_{h=RP+1}^{R} S(\mu_{\Delta\widetilde{\phi}_{kh}^{n*}})} \tag{8}$$

where S determines the areas of the fuzzy sets $\Delta\theta_{kh}^{n*}$ and $\Delta\widetilde{\phi}_{kh}^{n*}$, and their centroids are defined by $\overline{\theta}_{kh}$ and $\overline{\phi}_{kh}$. The crisp results $\Delta\theta_k^n$ and $\Delta\widetilde{\phi}_k^n$ are used to update the crisp values of the information-based control strategies as follows:

$$\theta_k^n = \theta_k^{n-1} + \Delta\theta_k^n \text{ and } \widetilde{\phi}_k^n = \widetilde{\phi}_k^{n-1} + \Delta\widetilde{\phi}_k^n \tag{9}$$

where θ_k^n represents the crisp proportion of drivers that must be recommended to take route k in iteration n, $\Delta\theta_k^n$ represents the adjustment of θ_k^n, $\widetilde{\phi}_k^n$ represents an associated crisp value for descriptive information, and $\Delta\widetilde{\phi}_k^n$ represents the adjustment of $\widetilde{\phi}_k^n$.

The centroid of $\mu_{\Delta\theta_{kh}^{n*}}$, which is the hth output fuzzy set for prescriptive information for route k, is defined as

$$\overline{\theta}_{kh} = \frac{\int y \cdot \mu_{\Delta\theta_{kh}^{n*}}(y) \cdot dy}{\mu_{\Delta\theta_{kh}^{n*}}(y) \cdot dy} \tag{10}$$

where y represents the domain of the membership function. After substituting (3) into the preceding equation, the centroid of each fuzzy set for prescriptive information can be defined in terms of its membership function parameters as follows:

$$\overline{\theta}_{kh} = \frac{\beta_{kh1}^{+}\left(3\cdot d_{kh1}+\beta_{kh1}^{+}\right)+\beta_{kh1}^{-}\left(3\cdot d_{kh1}-\beta_{kh1}^{-}\right)}{3\left(\beta_{kh1}^{+}+\beta_{kh1}^{-}\right)}. \tag{11}$$

Similarly, the centroid of each fuzzy set for descriptive information can be defined as follows:

$$\overline{\phi}_{kh} = \frac{\beta_{kh2}^{+}\left(3\cdot d_{kh2}+\beta_{kh2}^{+}\right)+\beta_{kh2}^{-}\left(3\cdot d_{kh2}-\beta_{kh2}^{-}\right)}{3\left(\beta_{kh2}^{+}+\beta_{kh2}^{-}\right)}. \tag{12}$$

Thus, the outputs of the fuzzy control model are completely defined in terms of the crisp inputs and the parameters of the membership functions, as required for the optimization approach discussed in Section IV.

D. Output Stage

Prescriptive information θ_k^n is directly used as output from the fuzzy control model. However, since descriptive information is linguistic, an additional step is required as follows to transform the continuous crisp value $\widetilde{\phi}_k^n$ to a discrete message:

$$\phi_k^\eta = \left\{\Phi_\omega \middle| \min_\omega \left(\left|\widetilde{\phi}_k^\eta - \overline{\Phi}_\omega\right| \qquad \forall\, \omega = 1, \ldots, 5\right)\right\} \tag{13}$$

where Φ_ω corresponds to "Very Light Traffic" if $\omega = 1$, "Light Traffic" if $\omega = 2$, "Moderate Traffic" if $\omega = 3$, "Heavy Traffic" if $\omega = 4$, and "Very Heavy Traffic" if $\omega = 5$. This is done by selecting the fuzzy set (linguistic message) with the closest centroid $(\overline{\Phi}_\omega)$ to $\widetilde{\phi}_k^n$. The rationality here is that the chosen fuzzy set has the largest degree of membership (or mapping) among all the possible fuzzy sets. In addition, the center of gravity method used to obtain (defuzzify) the outcome can be viewed as a measure of central tendency or weighted average, where the weights are the centroids of the fuzzy sets.

IV. Fuzzy Control Model Optimization Via H-Infinity Filtering

A. Error Function for Optimization

The optimization of the membership function parameters of the fuzzy control model can be viewed as a weighted least squares error minimization problem, where the error vector is the difference between the controller-estimated and SO proportions of drivers taking routes. This is done by using the derivatives of the weighted least squares error function Ω to solve the optimization problem. The expressions for these derivatives with respect to the half-widths and modal points of the membership functions can be obtained using (2)–(14). The derivative formulas for this type of error functions are given in [7]. The multidimensional error function is defined as follows:

$$\Omega_k = \frac{1}{2\cdot W\cdot N} \sum_{\sigma=1}^{W} \sum_{n=1}^{N} \lambda^n \left(E_k^{\rho(\sigma)n}\left(\theta_k^n, \phi_k^n\right) - \mathrm{SO}_k^{\rho(\sigma)}\right)^2 \tag{14}$$

where W is the number of stages of the rolling horizon approach, N is the number of iterations in a stage that the fuzzy control model uses to determine the information strategies, $E_k^{\rho(\sigma)n}$ are the controller-estimated proportions of drivers taking route k in the roll period of stage σ based on the information strategies (θ_k^n, ϕ_k^n) in iteration n of the search procedure, and λ^n is a weighting function. λ^n is defined here as N/n and weighs the initial iterations higher to induce faster convergence.

B. H-Infinity Filtering

As will be discussed hereafter, we apply the H-infinity filter to a nonlinear system defined by the membership function parameters of the fuzzy control model. The controller-estimated proportions of drivers taking routes constitute the output from this nonlinear system.

Consider the nonlinear time-invariant finite-dimensional discrete system defined by the inputs and outputs of the fuzzy membership functions of the fuzzy control model

$$X_k^{m+1} = f_X\left(X_k^m\right) + B \cdot w_k^m + \delta_k^m \tag{15}$$

$$\mathrm{SO}_k^{\rho(\sigma)} = h_X\left(X_k^m\right) + v_k^m \tag{16}$$

where m represents an iteration $(m = 1, \ldots, M)$ in the recursive process used to solve the filter (also called the "training" iteration), vector X_k^m is the state of the system for route k, $f_X(\cdot)$ is the identity mapping, $h_X(\cdot)$ is the nonlinear mapping between the fuzzy control model membership function parameters and their outputs in terms of the controller-estimated proportions of drivers taking route k ($E_k^{\rho(\sigma)}$), B is a tuning parameter, w_k^m and v_k^m are white noise sequences, and δ_k^m is an arbitrary noise sequence.

The augmented noise vector ε_k^m and the estimation error $\widetilde{x}_k^m$ are defined as follows:

$$\varepsilon_k^m = \left[[w_k^m]^T \ [v_k^m]^T \right]^T \tag{17}$$

$$\widetilde{x}_k^m = X_k^m - \hat{X}_k^m \tag{18}$$

where $\hat{X}_k^m$ is the estimate of X_k^m determined by the filter.

We now develop the expression for the system state. In Fig. 3, the fuzzy control model has five fuzzy sets for each input and each output. Following the definitions in Section III-B, the state of the nonlinear system is expressed as

$$X_k = \begin{bmatrix} b_{k11}^- \ b_{k11}^+ \ c_{k11} & \cdots & b_{k51}^- \ b_{k51}^+ \ c_{k51} \\ b_{k12}^- \ b_{k12}^+ \ c_{k12} & \cdots & b_{k52}^- \ b_{k52}^+ \ c_{k52} \\ \beta_{k11}^- \ \beta_{k11}^+ \ d_{k11} & \cdots & \beta_{k51}^- \ \beta_{k51}^+ \ d_{k51} \\ \beta_{k12}^- \ \beta_{k12}^+ \ d_{k12} & \cdots & \beta_{k52}^- \ \beta_{k52}^+ \ d_{k52} \end{bmatrix}^T . \tag{19}$$

The H-infinity filter finds $\hat{X}_k^m$ such that the infinity norm of transfer function G from the augmented noise vector ε to the estimation error $\widetilde{x}$ is bounded by a user-defined quantity α, i.e.,

$$\|G_{\widetilde{x}\varepsilon}\|_\infty < \alpha. \tag{20}$$

The estimate $\hat{X}_k^m$ is obtained through the following recursive H-infinity estimator [3]–[8]:

$$\begin{aligned} F_k^m &= \left.\frac{\partial f_X(X)}{\partial X}\right|_{X=\hat{X}_k^m} \\ H_k^m &= \left.\frac{\partial h_X(X)}{\partial X}\right|_{X=\hat{X}_k^m} \\ Q_k^0 &= E\left(X_k^0 \cdot [X_k^0]^T\right) \\ Q_k^m &\left(I - [H_k^m]^T \cdot H_k^m \cdot P_k^m\right) = \left(I - Q_k^m/\alpha^2\right) P_k^m \\ Q_k^{m+1} &= F_k^m \cdot P_k^m \cdot [F_k^m]^T + B \cdot B^T \\ K_k^m &= F_k^m \cdot P_k^m \cdot [H_k^m]^T \\ \hat{X}_k^{m+1} &= F_k^m \cdot \hat{X}_k^m + K_k^m \left(\mathrm{SO}_k^\sigma - h_X\left(\hat{X}_k^m\right)\right) \end{aligned} \tag{21}$$

where B and α are tuning parameters proportional to the magnitudes of the artificial noise processes. Q and P are assumed to be nonsingular sequences of matrices. K is known as the H-infinity gain. Q^0 is the initial state covariance matrix. H is the partial derivative of the error function Ω with respect to the membership function parameters, and F is the identity matrix.

Existing implementations of the H-infinity filter to optimize the membership function parameters of a fuzzy control model consider a single desired system state. By contrast, here, the filtering approach is implemented, considering multiple desired states. This increases the complexity of the problem as the same $\hat{X}_k^m$ needs to enable the effective and efficient determination of the information strategies for several system states.

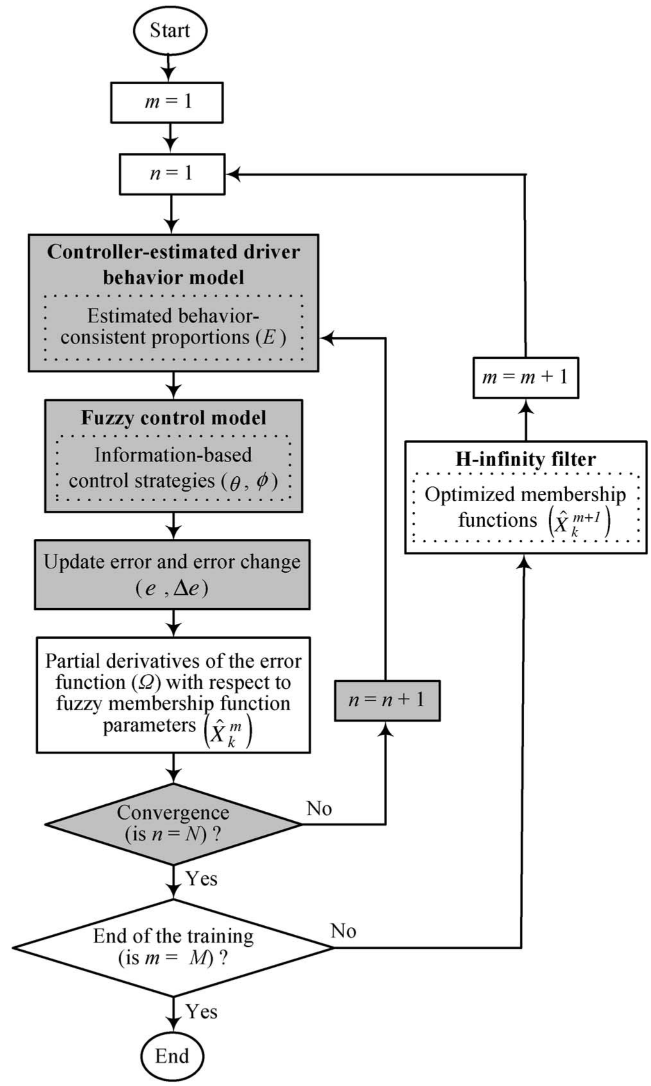

Fig. 4. Filter-based optimization of fuzzy control model parameter.

After obtaining $\hat{X}_k^m$, they are used by the fuzzy control model to determine the information strategies (θ_k, ϕ_k). In summary, tuning the parameters of the fuzzy control model leads to faster convergence to the information strategies.

C. Recursive Solution Scheme

Fig. 4 conceptually illustrates the implementation of the recursive approach used to solve the H-infinity filter discussed here. It consists of an inner loop that is an extended version of the iterative search procedure illustrated in Fig. 2.

The additional component is the step used to calculate the partial derivatives of the error function Ω with respect to the current estimated vector $\hat{X}_k^m$ of membership function parameters. After the extended search procedure is completed (when $n = N$), updated values of the derivatives are obtained, which are then used in the H-infinity estimator (21) of the outer loop to determine new values for the fuzzy membership function parameters. This recursive scheme of the outer loop is repeated

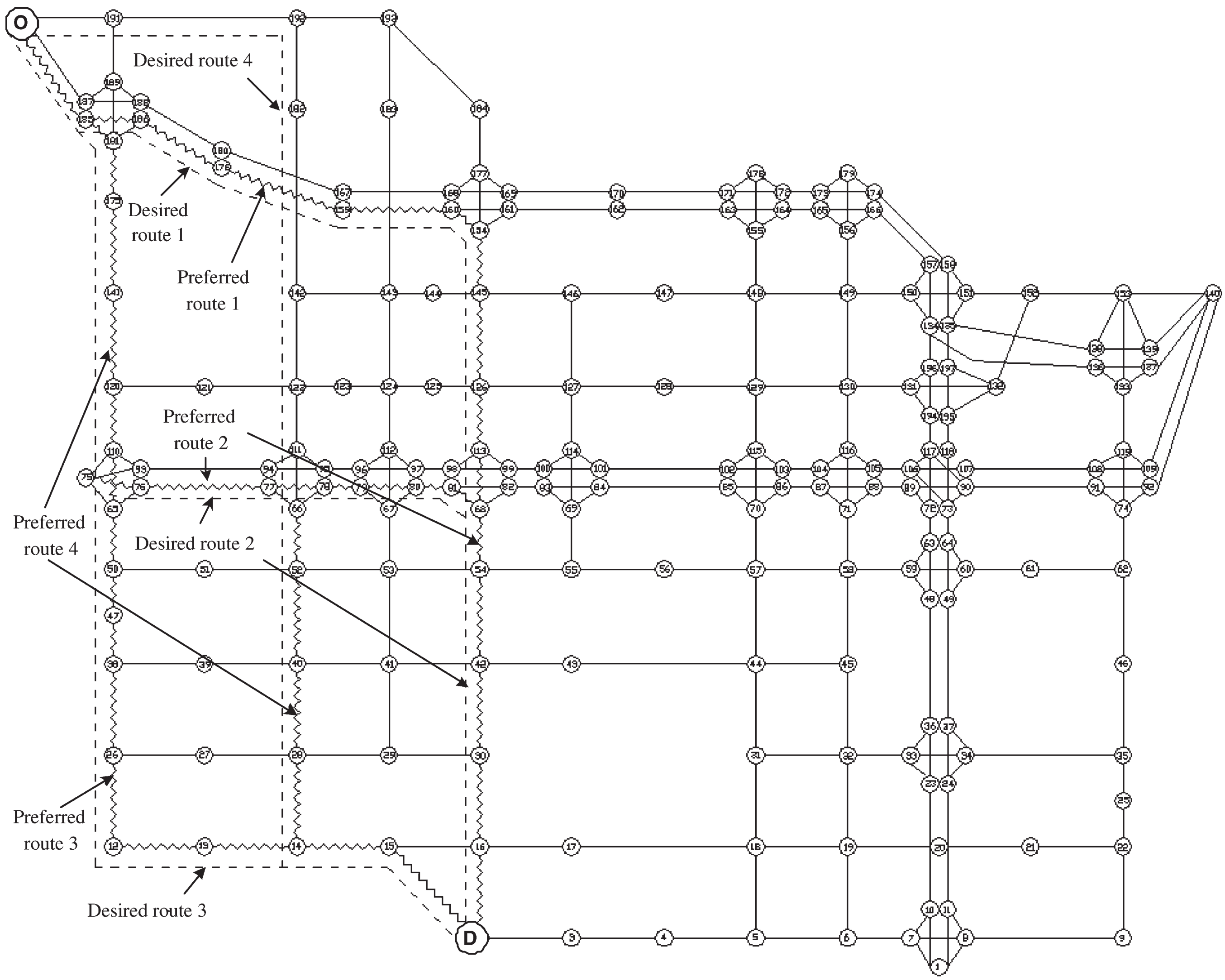

Fig. 5. Borman expressway corridor network.

until M training iterations are completed. The outcome of the recursive scheme is the set of optimized membership function parameters.

V. EXPERIMENTS

Two sets of simulation experiments—one at the O-D pair level and the other at the network level—are conducted to illustrate the advantages of using optimized fuzzy control model parameters to determine the information-based control strategies. In all experiments, it is assumed that all drivers have capabilities to receive personalized information.

A. Network Characteristics

The experiments are conducted for the Borman expressway corridor network shown in Fig. 5. Located in northwest Indiana, it consists of a 16-mi section of I-80/94 (called the Borman expressway), I-90 toll freeway, I-65, and the surrounding arterials and streets. It has 197 nodes and 460 links. The Borman expressway is a highly congested freeway with a large fraction of semitrailer truck traffic. To manage traffic at these high congestion levels, particularly under incidents, an advanced traffic management system has been installed on the Borman network to provide drivers with real-time traffic information. The Indiana toll road, I-90, which operates parallel to the Borman expressway, is a potential alternative for diverting traffic. Depending on the destination, other potential major alternative routes exist as well (such as US-20, US-30, Ridge Road, and 73rd Avenue).

B. Convergence Criteria

The optimized fuzzy control model is benchmarked against the default model by comparing the performance of its search procedure with that of the default search procedure. The convergence criterion used to evaluate the performance of both the default and optimized fuzzy control models is

$$\sqrt{\frac{1}{5}\left(\sum_{n=A-4}^{A}\left(e_k^n - \widetilde{e}_k^A\right)\right)^2} < \rho \qquad (22)$$

TABLE II
DECISION (IF–THEN) RULES FOR THE RULE -BASED CONTROLLER -ESTIMATED DRIVER BEHAVIOR MODEL

Category	LHS	RHS
Travel time (*T*)	If *T* is Very Low (VL)	Driver will choose the alternative (O)
	If *T* is Low (L)	Driver will probably choose the alternative (PO)
	If *T* is Medium (M)	Driver is indifferent to the alternative (I)
	If *T* is High (H)	Driver probably will not choose the alternative (PN)
	If *T* is Very High (VH)	Driver will not choose the alternative (N)
Route complexity (*NN*)	If *NN* is Very Low (VL)	Driver will choose the alternative (O)
	If *NN* is Low (L)	Driver will probably choose the alternative (PO)
	If *NN* is Medium (M)	Driver is indifferent to the alternative (I)
	If *NN* is High (H)	Driver probably will not choose the alternative (PN)
	If *NN* is Very High (VH)	Driver will not choose the alternative (N)
Prescriptive information (*Y*) for more responsive drivers	If *Y* is Route is Recommended (RR)	Driver will choose the alternative (O)
	If *Y* is Route is Not Recommended (RNR)	Driver will not choose the alternative (N)
Prescriptive information (*Y*) for less responsive drivers	If *Y* is Route is Recommended (RR)	Driver will probably choose the alternative (PO)
	If *Y* is Route is Not Recommended (RNR)	Driver probably will not choose the alternative (PN)

where A is the number of current iterations of the fuzzy control model, $\tilde{e}_k^A$ is the average value of the error over the last five iterations for route k, e_k^n is the error in iteration n for route k, and ρ is a prespecified small constant that indicates the required accuracy. Convergence is achieved when the inequality in (22) is satisfied for all controllable routes for a given O-D pair.

C. Controller-Estimated and Actual Driver Behavior

Drivers typically perceive/identify a set of preferred routes that they would likely consider for the current day based on their past experience and current network conditions (provided through advanced information systems). It is reasonable to assume [9] that, in an actual deployment context, the controller can estimate the set of preferred routes for each driver through driver surveys, historical traffic data, wide-area traffic surveillance systems, tracking systems, Global Positioning System data, and/or two-way communication systems. However, it is unrealistic to expect that the controller knows exactly the specific preference that a driver assigns to each route in the preferred set and the effect of information on the associated decision-making process. To ensure deployment realism in terms of data availability and consistency with likely driver behavior, in our approach, the controller uses an estimated driver behavioral model to determine the probability that a driver chooses a specific route from his/her preferred set.

It should be noted here that, in reality, a driver's behavioral model can be different from the one assumed by the controller. In our experiments, it is assumed that they are different so as to obtain more conservative performance estimates. The controller-estimated driver behavior model is a nonlinear rule-based route choice model (Table II).

The *actual* behavior of drivers is assumed to be based on a random coefficient path-size multinomial logit model. Equation (23) shows the structure of the behavioral model used by the drivers. The path-size component corresponds to the model proposed by Ben-Akiva and Bierlaire [10] and Ramming [11]. The objective of the path-size component is to account for the effect of links being common to different routes. Thus, the path-size factor is an approximated measure of the amount of overlap of an alternative with all other alternatives in the route choice set. Ignoring the effect of link overlaps across the set of alternatives can potentially result in high and unrealistic volumes over the set of common links.

We assume here that the distributions of the coefficients β of the driver behavioral model are identical across all drivers. However, as illustrated in (23), these coefficient values vary across individual drivers to represent random taste variations across drivers. This is designed to increase modeling realism. The random coefficient path-size multinomial logit model used to represent the actual driver route choices is given as follows:

$$U_k^r = \beta_{ET}^r \cdot ET_k^r + \beta_C^r \cdot C_k + \beta_Y^r \cdot Y_k^r + \beta_{PS}^r \cdot \ln(PS_k) + \varepsilon_k^r \tag{23}$$

where

PS_k $= \sum_{a\in\Gamma_k}(l_a/L_k)(1/\sum_{j\in PK}((L_k^\lambda/L_j^\lambda)\cdot\delta_{aj}))$ path-size component for route k;
U_k^r utility of route k for driver r;
β_x^r coefficient of variable/function x for driver r;
ET_k^r expected travel time on route k for driver r;
C_k number of nodes on route k;
Y_k^r route recommendation for driver r, which has a value of 1 if route k is recommended and 0 otherwise;
ε_k^r independent identically distributed extreme value disturbance or random component for route k and driver r;
Γ_k set of links on route k;
l_a length of link a, $a \in \Gamma_k$;
L_j length of route j;
PK set of routes considered (preferred) by the driver when making a route choice decision;
λ model parameter;
δ_{aj} link-route incidence dummy, which has a value of 1 if route j uses link a and 0 otherwise.

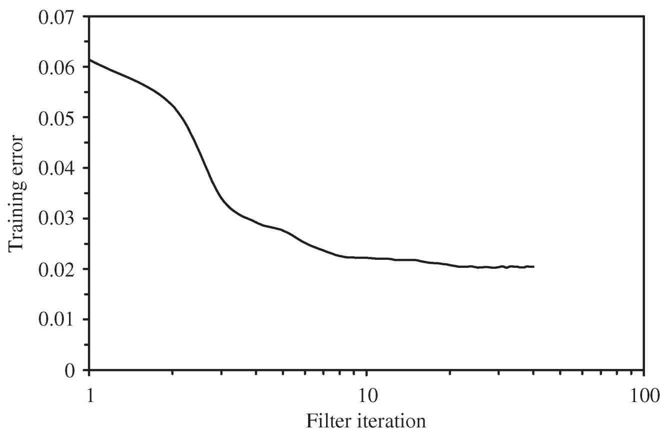

Fig. 6. Training progress of the filtering approach.

The controller-estimated driver behavior model is initially calibrated using the output from the actual driver behavior model as a proxy for field measurements. As mentioned earlier, it is important to highlight here that the controller does not use any information about the actual driver behavior model. Furthermore, the proposed approach is independent of the structure of the controller-estimated driver behavior model, providing a modular capability to use any driver behavior model structure.

D. Experiments: O-D Level

In these experiments, a single O-D pair is used to illustrate methodological insights of the proposed H-infinity-filter-based optimized fuzzy control model. As shown in Fig. 5, there are four driver-preferred routes (zigzag lines) and four controller-desired routes (dashed lines) connecting the selected O-D pair, but only three of them fully overlap. The controller seeks to achieve the SO proportions only for the set of controllable routes (the three routes that fully overlap). Thus, routes 1, 2, and 3 are defined as the controllable routes, with the corresponding SO proportions being 44%, 33%, and 11%, respectively.

Fig. 6 depicts the progress of the training of the fuzzy membership function parameters for this O-D pair using the H-infinity filtering approach. At the end of the process, the filter is able to capture a significant portion of the error. The consequence of this is illustrated in Fig. 7, where the trajectories of the controller-estimated proportions of drivers choosing routes are depicted under the optimized and default search procedures. In the first iteration, the controller recommends routes based on the SO proportions. Each iteration of the search procedure seeks information strategies (BC proportions), so that the controller-estimated proportions of drivers choosing controllable routes get closer to the corresponding SO proportions. When the search procedure converges, the corresponding BC proportions are recommended to drivers. The results show a significant reduction in the number of iterations that is required to achieve convergence under the optimized fuzzy control model. In this example, the convergence of the optimized search procedure requires only eight iterations, whereas the default search procedure requires 15 iterations. It represents an improvement of more than 45%.

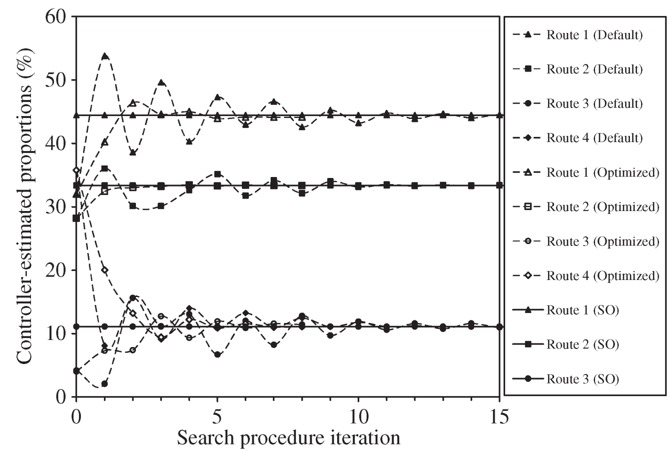

Fig. 7. Trajectory of the controller-estimated proportions of drivers choosing routes for the default and optimized fuzzy control models.

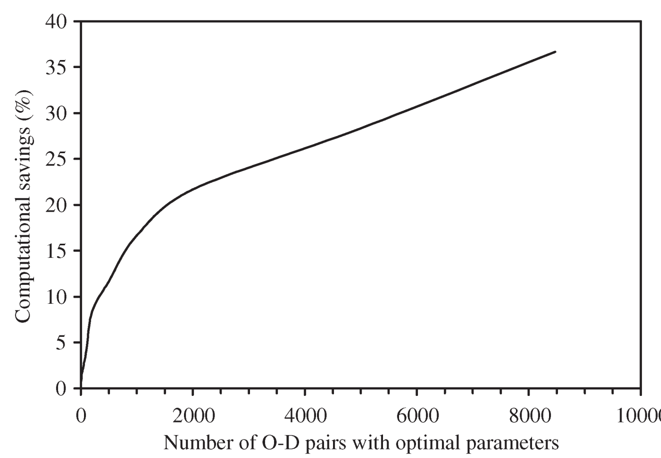

Fig. 8. Computational savings relative to the default fuzzy control model.

E. Experiments: Network Level

These experiments evaluate the performance and effectiveness of the H-infinity filter at the network level. They are conducted using the broader framework of the BC real-time traffic routing problem illustrated in Fig. 1. In the experiments, the stage length is 20 min, and the roll period is 5 min.

Fig. 8 depicts the computational savings under the optimized parameters relative to the default search procedure for various numbers of O-D pairs. It emphasizes the significant value in using optimized parameters. The computational requirements increase with the number of controllable routes, the length (number of nodes) of the routes, and the associated volumes. Hence, the number of O-D pairs is not a direct proxy for computational load. This aspect is illustrated in the figure, where the rate of increase in computational savings is initially higher, as the associated O-D pairs tended to be mostly connected by controllable routes and had higher traffic volumes. As more O-D pairs use optimized parameters, the likelihood of more noncontrollable routes being considered is higher as is the possibility of controllable routes with lower traffic volumes. Hence, for some of the additional O-D pairs, the effect of using the optimized parameters is not as large, leading to the constant rate of increase as the number of O-D pairs increase beyond 2000.

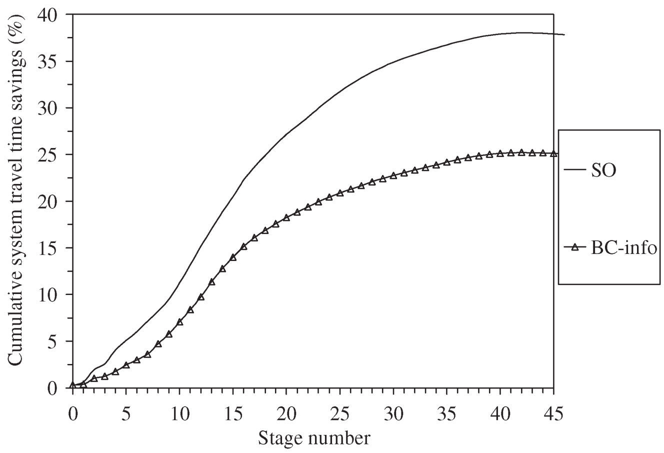

Fig. 9. Cumulative system travel time savings benchmarked against the no-information scenario.

Fig. 9 shows the cumulative system travel time savings under the ideal SO and the BC real-time traffic routing (BC-info) strategies relative to the no-information scenario. By definition, the SO strategy has the highest cumulative system travel time savings. Hence, it represents the benchmark for comparing the performance of other strategies. The BC-info strategy results in significant improvements to the system performance relative to the no-information scenario. This figure is shown here to indicate that the use of the H-infinity filter does not adversely affect the capability of the (optimized) fuzzy control model to determine BC traffic routing strategies and results in savings identical to those under the default fuzzy control model.

VI. Conclusion

This paper shows the effectiveness of the proposed offline H-infinity filtering scheme to optimize the membership function parameters of the fuzzy control model used in the approach to generate BC traffic routing strategies for real-time deployment. The optimized fuzzy membership function parameters can better respond to nonlinearities and noise uncertainty, leading to enhanced computational performance. This ability to significantly improve computational efficiency is critical from a practical standpoint, because the information strategies are required in subreal time under the rolling horizon deployment approach.

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